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Probing neutron star superfluidity with gravitational-wave data

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We discuss the possibility that future gravitational-wave detectors may be able to detect various modes of oscillation of old, cold neutron stars. We argue that such detections would provide unique insights into the superfluid nature of neutron star cores, and could also lead to a much improved understanding of pulsar glitches. Our estimates are based on a detector configuration with several narrowbanded (cryogenic) interferometers operating as a "xylophone" which could lead to high sensitivity at high frequencies. We also draw on recent advances in our understanding of the dynamics of pulsating superfluid neutron star cores.

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The extremes of physics. — Since their discovery in the late 1960s neutron stars have emerged as unique probes of many extremes of physics. Comprising roughly one and a half solar masses of material compressed inside a radius of ten kilometers, i.e well beyond nuclear density, the neutron stars still hide many of their mysteries. For example, we do not yet fully understand the nature of the supranuclear equation of state required to describe matter in the core of neutron stars. Still, a picture has emerged in which superfluidity plays a vital role. Theoretically, such a picture has as its foundation [1] the extraordinarily successful many body theory of Fermi liquids and the BCS mechanism that are used to describe superconductors and superfluid Helium three. It is now generally believed that once a neutron star cools below a few times 10^9 K (a few months after its birth) the bulk of its core will become superfluid. Thus, the more than 1000 observed pulsars provide useful laboratories for studying large scale superfluidity and truly high T_c superconductors.

The main observational evidence for neutron star superfluidity is provided by the glitches (a sudden spin-up followed by a long term relaxation period) that have been observed in roughly 30 pulsars. For a summary of the current results see Table 5 in [2]. While the smaller glitches in, for example, the Crab pulsar can be understood in terms of spindown induced quakes in the neutron star's crust [3], this model cannot explain the large glitches seen in, for example, the Vela pulsar [4]. In the long favoured model for large glitches the observed spin-up is caused by the transfer of angular momentum from a superfluid that coexists with lattice nuclei in the inner crust (extending from neutron drip to $\sim 2\times 10^{14}$g/cm^3). This idea is based on the notion that the vortices by means of which the superfluid "rotates" can be pinned to the crust and therefore be prevented from moving outwards (and thus spinning the superfluid down) as the crust undergoes magnetic braking. Once sufficient strain between the two components has been built up the vortices undergo chaotic "unpinning" and a glitch occurs [4]. However, very recent observations of possible free precession in PSR B1828-11 [5] seem to indicate a much lower degree of superfluid vortex pinning than is usually assumed in the glitch models [6]. Thus the standard glitch model would seem to be in trouble and our need for an improved understanding of neutron star superfluidity and its astrophysical manifestation is amply illustrated.

"Gravitational-wave asteroseismology". — The exciting possibility that gravitational waves from pulsating neutron stars may prove to be detectable, and that a knowledge of the various mode frequencies will provide strong constraints on the supranuclear equation of state has been discussed in a series of papers [7, 8, 9]. However, the most recent estimates suggest that such a detection with (say) LIGO II requires mode-excitation to possibly unrealistic amplitudes [8]. The most promising scenario corresponds to the various modes being excited following the formation of the neutron star after gravitational collapse. This could potentially lead to an energy equivalent to $10^{-6}M_\odot c^2$ being radiated as gravitational waves, and there is no obvious reason why a comparable energy should not be deposited in various nonradial modes of oscillation. But there are still two problems with this scenario: Firstly, the event rate would be rather low unless the waves from neutron stars born in the Virgo supercluster (at a distance of roughly 20 Mpc) can be seen (and this seems unlikely [8]). Secondly, it may be difficult to detect the associated gravitational waves even from events in our own galaxy should significantly less energy be radiated. Basically, the recent estimates suggest that the detectability of the waves using an advanced LIGO detector is likely to be marginal even from the most optimistic astrophysical scenarios. On the other hand, the possibility that such observations may help shed light on the true nature of matter at extreme densities provides strong motivation for pursuing work in this direction. It also seems likely that the first direct detection of gravitational waves will be the initial step towards a true revolution in the way that we view the Universe, leading to the development of detectors with significantly increased sensitivity. We feel that — as a new generation of detectors is about to come on-line — it is appropriate to speculate about future challenges for this field.

The various gravitational-wave detector groups are already discussing possible technological improvements that may be achievable in the future. As an example of a suitably advanced instrument we will take the so-called EURO detector, for which the noise-level has been estimated by Sathyaprakash and Schutz (for further details see [10]). We will consider two possible configurations: In the first, the sensitivity at high frequencies is limited by the photon shot-noise, while the second configuration reaches beyond this limit by running several narrowbanded (cryogenic) interferometers as a "xylophone". The corresponding noise-curves are illustrated, and compared to the current generation of interferometers, in Figure 1.

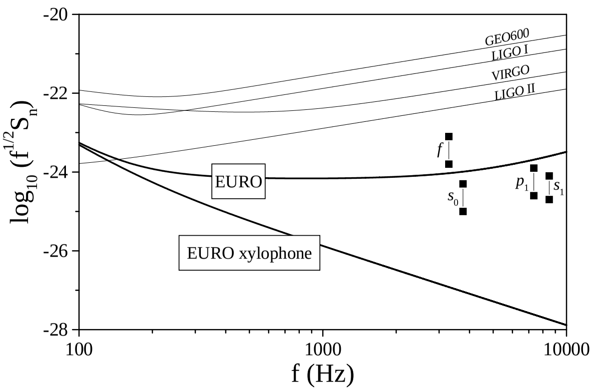

FIG. 1: The spectral noise density for the new generation of laser-interferometric gravitational-wave detectors that will come online in the next few years (thin lines) is compared to the more speculative estimates for the EURO detector (solid lines). A key feature of this advanced configuration is that it may operate several narrowbanded interferometers as a xylophone, thus reaching high sensitivity at khz frequencies. We also indicate the effective gravitational-wave amplitudes from the glitch-induced mode-oscillations discussed in the text.

By comparing the results of previous discussions of gravitational-wave asteroseismology to the data in Figure 1 we immediately see that a EURO detector would provide a superb instrument for studying pulsating neutron stars. This means that previously suggested strategies [8] for unveiling the supranuclear equation of state may be put to the test. However, we want to be more ambitious than this. We also want to be able to infer the parameters of neutron star superfluidity and possibly shed light on the mechanism for pulsar glitches.

Superfluid neutron star seismology. — An ordinary fluid neutron star has a plethora of pulsation modes, the most familiar of which are the pressure p-modes (the fundamental of which is known as the f-mode) and the gravity g-modes. In addition to these families of modes, the non-zero shear modulus in the neutron star crust provides support for families of modes, the Coriolis force leads to the presence of the so-called r-modes in a rotating star et cetera (see [11] for a concise description). Basically, each extra piece of physics of a detailed neutron star model brings (at least) one new family of non-radial modes into play. So what happens as the core of the star becomes superfluid?

In the simplest description, a superfluid neutron star core can be discussed in terms of two distinct fluids. One of these fluids represents the superfluid neutrons and the other fluid represents a charge-neutral conglomerate of the remaining components (which are expected to be coupled on a relatively short timescale). The fact that these two fluids — which we will refer to as the "neutrons" and the "protons" — can flow more or less independently provides one of the main distinguishing dynamical features of a superfluid neutron star. The neutrons and the protons are coupled via the so-called entrainment effect, whereby the momentum of one constituent carries along part of the mass of the other constituent (analogous to the way that the momentum of one of the constituents is thought to carry along (or entrain) some of the mass of the other in a mixture of the two superfluids Helium three and Helium four [12]). A superfluid is locally irrotational, but it mimics large-scale rotation by forming a dense array of quantized vortices. Because of the entrainment effect the flow of neutrons around these vortices will induce a flow also in a fraction of the protons. This leads to magnetic fields being attached to the vortices and because of the electromagnetic attraction of the electrons to the protons (the timescale of which is very short) there will then be a dissipative scattering of the electrons. This dissipative mechanism is known as mutual friction, and since it serves to damp out any relative motion between the neutrons and the protons it is expected to be relevant both for models of the spin-evolution of a pulsars [13] and discussions of mode-oscillations [14].

However, the crucial parameters of neutron star superfluidity are not very well determined. For example, the strength of mutual friction depends crucially on the ratio between the "bare" and "effective" proton masses m_p and m_p^*. Estimates of this ratio suggest that $0.3 \le m_p^*/m_p \le 0.8$. Clearly, any observation that could help determine this ratio (and constrain other relevant parameters) must be seen as an attractive proposition. This prompts the main question motivating this letter: Can gravitational-wave asteroseismology provide us with some of the desired answers? As a first step towards answering this question we need to understand the nature of the pulsation modes of a superfluid neutron star core, and the effect that entrainment has on the spectrum of the emerging gravitational waves.

In a recent study [15], aimed at comparing and contrasting oscillations of normal and superfluid neutron star cores, we have discussed this issue in detail. Briefly, our main conclusions are as follows: There are two sets of (predominately acoustic) pulsation modes in a superfluid core. Both mode-families are associated with high-frequency modes and the oscillation frequencies are interlaced in the spectrum. One set of modes is the familiar p-modes, for which the two fluids tend to move together. The other set of modes (exemplified by s_0 and s_1 in Table I) is distinguished by the fact that the protons and neutrons are largely "countermoving". The existence of these two classes of modes has been established in several studies, but their true nature and the relation to the normal fluid case has not been clearly explained previously. Of particular interest for our current discussion is the fact that the superfluid set of modes is strongly dependent on the entrainment parameters. From the relevant local dispersion relation (see [15] for a complete discussion) one finds that superfluid modes can be qualitatively represented by the following, approximate, solution for the local mode-frequency:

$$\omega_s^2 \approx \frac{m_p}{m_p^*}\frac{l(l+1)}{r^2}c_p^2 \ , \tag{1}$$

where c_p^2 is (roughly) the sound speed in the proton fluid, r is the radial coordinate, and l is the index of the relevant spherical harmonic $Y_{lm}(\theta,\varphi)$ used to describe the angular dependency of the mode. From this relation it is clear that an observation of these modes would provide potentially unique information regarding the nature of large scale superfluidity.

We have recently performed the first ever (fully relativistic) calculations of both frequency and damping rate of the oscillation modes of a superfluid neutron star [16]. This calculation clearly distinguishes the two families of pulsation modes, and highlights the fact that the superfluid mode frequencies are strongly dependent on the parameters of entrainment (mainly the average value of the effective proton mass m_p^*, cf. (1)). The first few mode-frequencies and the associated gravitational-wave damping times for one of our model stars are listed in Table I. (We note in passing that some authors [17] have suggested that the superfluid modes will not radiate gravitational waves.) The given mode-results were obtained for a non-rotating stellar model and a simple equation of state corresponding to a combination of two polytropes. These results should provide a good "order of magnitude" insight into future results for realistic supranuclear equations of state. In particular, since most observed pulsars are slowly rotating (in the sense that they spin at a fraction of the mass-shedding limit) these results should be relevant for all but the fastest millisecond pulsars. Likewise, Mendell [18] has shown that two-fluid (neutron and proton) models of neutron star superfluidity (where magnetic fields, vortex pinning effects, etc. are ignored) are reasonable when the core matter oscillations are in the kilohertz range.

Unveiling the nature of pulsar glitches. — To what extent will future gravitational-wave observations be able to detect the various pulsation modes of a superfluid neutron star? Let us assume that a typical gravitational-wave signal from a neutron star pulsation mode takes the form of a damped sinusoidal, i.e.

$$h(t) = \mathcal{A}e^{-(t-T)/t_d}\sin[2\pi f(t-T)] \quad \text{for } t > T \tag{2}$$

where T is the arrival time of the signal at the detector (and $h(t)=0$ for $t<T$). Using standard results for the gravitational-wave flux [8], the amplitude $\mathcal{A}$ of the signal can be expressed in terms of the total energy radiated through the mode:

TABLE I: The frequency and damping rate for the first few modes of Model II of Comer et al [16]. The corresponding star has mass $1.36 M_\odot$ and radius 7.9 km, and could be considered as a "reasonable" model for a superfluid neutron star core. We also show the gravitational-wave signal-to-noise ratios resulting from the glitch model discussed in the main text. The results correspond to an advanced EURO detector with (model 1) and without (model 2) photon shotnoise, respectively. The lower estimate is for a Crab glitch while the upper estimate follows from the Vela data.

Mode	f (kHz)	t_d (s)	Model 1	Model 2
f	3.29	0.092	0.4—6	300—4.7×10^3
p_1	7.34	1.01	0.08—1.2	680—1×10^4
s_0	3.76	15.75	0.3—4.8	350—5.4×10^3
s_1	8.49	1.29	0.06—0.9	780—1.2×10^4

$$\mathcal{A} \approx 7.6\times10^{-24}\sqrt{\frac{\Delta E_\odot}{10^{-12}}\frac{1\ \text{s}}{t_d}}\left(\frac{1\ \text{kpc}}{d}\right)\left(\frac{1\ \text{kHz}}{f}\right) \ . \tag{3}$$

where $\Delta E_\odot = \Delta E/M_\odot c^2$. Finally, the signal-to-noise ratio for this signal can be estimated from [8]

$$\left(\frac{S}{N}\right)^2 = \frac{4Q^2}{1+4Q^2}\frac{\mathcal{A}^2 t_d}{2S_n} \tag{4}$$

where the "quality factor" is $Q=\pi f t_d$ and S_n is the spectral noise density of the detector (in Figure 1 we show the dimensionless strain $\sqrt{f S_n}$ for various detector configurations).

The main question here is: What amount of energy should one assume to be channeled through the various modes? In the previous studies it was assumed that as much as $10^{-5}M_\odot c^2$ could be radiated [8]. However, this number is likely only relevant (if at all) for the oscillations of the remnant following a strongly asymmetric gravitational collapse. It is not useful for our present considerations since such a nascent neutron star will be hot (above 10^{10} K) enough for the core not to be superfluid. For a reasonable scenario, we turn to the indications that young neutron stars are seismically active. As a suitably simple model scenario we will assume that oscillations in the superfluid core are excited following a glitch. The released energy can then be estimated from

$$\Delta E \approx I\Omega\Delta\Omega \approx (10^{-6}-10^{-8})I\Omega^2 \tag{5}$$

where $\Omega = 2\pi/P$. In this formula it is appropriate to use the moment of inertia $I\sim10^{45}$ gcm^2 of the entire star, since the spin-up incurred during the glitch remains on timescales that are much longer than the estimated coupling timescale between the crust and the core fluid. Using this formula, and the data for the Crab and Vela pulsars (see Table II) we can estimate the energy associated with typical glitch events. (We note that these estimates are similar to ones already in the literature [3, 19].)

Assume that a comparable amount of energy goes into exciting oscillations in the core superfluid. This is obviously ad hoc, but it provides a reasonable order-of-magnitude starting point for this kind of discussion. Using the spectral density estimated for the EURO detector we then readily estimate the associated signal-to-noise ratio from (4). The results of this exercise are listed in Tables I-II. From this data we can see that the various modes would be marginally detectable given this level of excitation and a third generation detector limited by the photon shotnoise. But if this limit can be surpassed by configuring several narrowbanded interferometers as a xylophone, the achievable signal-to-noise ratio is excellent. Besides estimating the signal-to-noise ratio for the various oscillation modes for a given radiated energy, we can ask related questions relevant for the inverse problem. For example, we can confirm that the oscillation frequencies can be extracted with good accuracy from the data. This then enables us to distinguish clearly between the "normal fluid" f and p-modes and the superfluid s-modes. In other words, we would have the information required not only to infer the mass and radius of the star [8], we could also hope to constrain the parameters of neutron star superfluidity. In addition to providing this important information, these observations could provide a unique insight into the glitch mechanism itself since the multipolar structure of the excited modes reflects the symmetry (or lack thereof) of the triggering mechanism.

TABLE II: Data for archetypal glitching pulsars.

PSR	P (ms)	d (kpc)	$\Delta\Omega/\Omega$	$\Delta E/M_\odot c^2$
Crab	33	2	10^{-8}	2×10^{-13}
Vela	89	0.5	10^{-6}	3×10^{-12}

Issues. — This is a very exciting time for gravitational physics. The opening of a new window to the Universe may well lead to a fundamental change in the way that we view Nature. For example, it is generally expected that gravitational-wave observations will provide us with crucial information regarding the details of neutron star physics, eg. by constraining the supranuclear equation of state. Such information may to a certain extent be obtained by analysing signals from inspiraling neutron star binaries, see for example [20]. As we have argued in this Letter an observation of the various modes of oscillation of (say) a glitching pulsar would provide a truly unique probe of the internal physics that could help improve our understanding of, in particular, large-scale superfluidity. The potential precision of this method easily surpasses other proposed methods for studying neutron star superfluidity, eg. by observed cooling data [21]. Of course, it seems likely that our proposed scheme requires the development of detectors with sensitivity beyond the goals set for the advanced LIGO configuration. However, it is clear that, even though the construction of a gravitational-wave detector with the sensitivity of EURO provides a serious challenge, a successful effort in this direction would be richly rewarded.

If we are to achieve this ambitious goal we must also make significant progress on the theoretical side. We need dynamical studies of general relativistic superfluid neutron stars comprising a crust in the outer parts and other possible exotic phases of matter in the core. We need to improve our understanding of the mechanisms that couple the various components of a realistic neutron star, and estimate the relevant coupling timescales. We need numerical simulations that investigate the extent to which the various oscillation modes are excited by plausible mechanisms. These, and related (eg. regarding rapidly spinning stars and possible mode-instabilities [22]), topics are tremendously exciting since they challenge our understanding of the very extremes of physics.

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